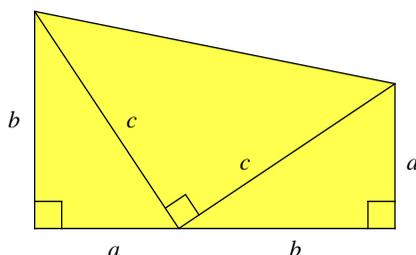


Proof Without Words: President Garfield and the Cauchy–Schwarz Inequality

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It is well known (see, for example, [1]) that James A. Garfield, the twentieth President of the United States, published a proof of the Pythagorean theorem in 1876, five years before he became President. His proof is based on the yellow trapezoid below, and proceeds by computing its area in two different ways.

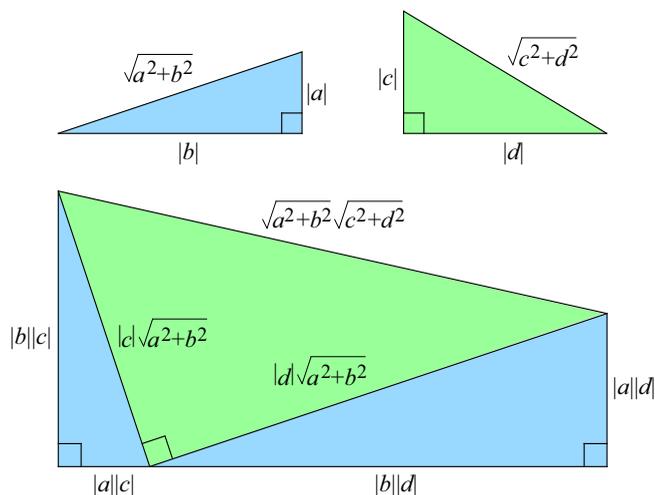


Although there is absolutely no evidence that he did so, Garfield could have used another trapezoid consisting of three right triangles to prove wordlessly the two-dimensional version of the Cauchy–Schwarz inequality.

Theorem. For real numbers a , b , c , and d ,

$$|ac + bd| \leq \sqrt{a^2 + b^2} \sqrt{c^2 + d^2}.$$

Proof.



$$|ac + bd| \leq |a| |c| + |b| |d| \leq \sqrt{a^2 + b^2} \sqrt{c^2 + d^2}. \quad \blacksquare$$

REFERENCE

1. H. Eves, *Great Moments in Mathematics (Before 1650)*. Mathematical Association of America, Washington, 1980.

Summary. We prove wordlessly the Cauchy–Schwarz inequality (for $n = 2$) using a trapezoid partitioned into three right triangles.

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**From the Files of Past MAGAZINE Editors
J. Arthur Seebach and Lynn Arthur Steen 1976–1980**

Lynn Steen’s favorite articles in the MAGAZINE during his tenure as co-editor were by P. Halmos, Logic from A to G, *Math. Mag.* **50** no. 1 (1977) 5–11 and V. Klee, Some unsolved problems in plane geometry, *Math. Mag.* **52** no. 3 (1979) 131–145.



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When Lynn Arthur Steen (LAS) and J. Arthur Seebach, Jr. (AS) were co-editors of the MAGAZINE, submissions were sent via post and were typewritten manuscripts. When an article was accepted, LAS and AS would mark changes on the typewritten manuscript and then LAS, AS, and the author would receive a typeset version. All would read the manuscript and look for corrections. Because corrections were coming from several different directions at the same time, sometimes they were mixed up, like the time the word “number” was used when “integer” would have been correct. One person said “insert ‘whole before ‘number’,” another said “change ‘number’ to ‘integer’,” both changes were made resulting in “whole integer.”