# Proof Without Words: Ptolemy's Inequality

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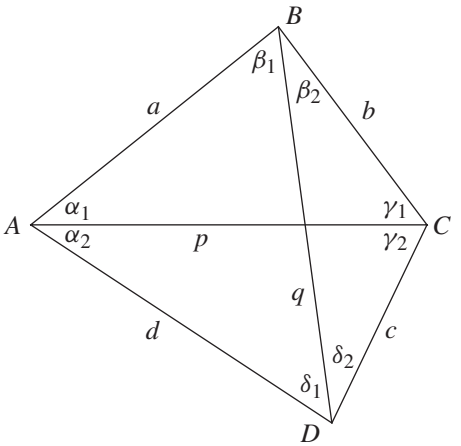
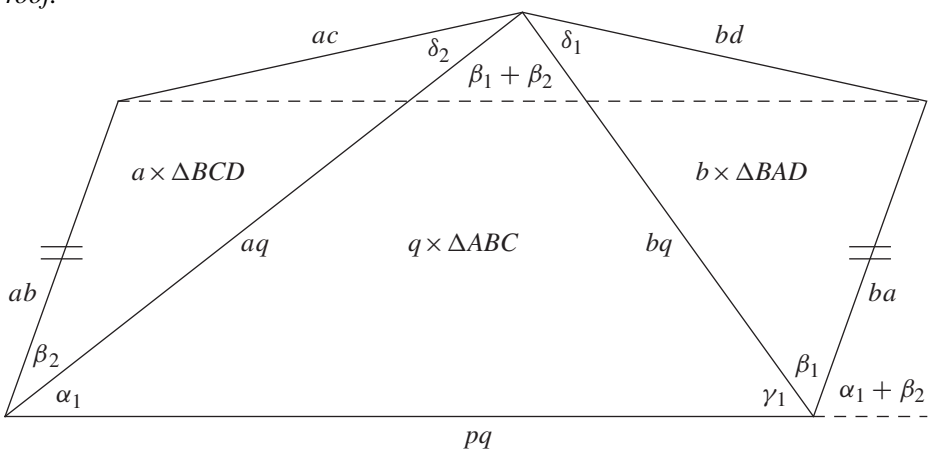



PTOLEMY'S INEQUALITY. In a convex quadrilateral with sides of length $a, b, c, d$ (in that order) and diagonals of length $p$ and $q$, we have $pq \le ac + bd$.

*Proof.*




NOTE. The angle at the top of the figure, $\delta_2 + \beta_1 + \beta_2 + \delta_1$, is drawn as being smaller than $\pi$, but the broken line representing $ac + bd$ is at least as long as the base of the parallelogram in any case. In a cyclic quadrilateral, pairs of opposite sides have sum $\pi$ so that $\delta_2 + \beta_1 + \beta_2 + \delta_1 = \pi$, leading to equality:

PTOLEMY'S THEOREM. In a cyclic quadrilateral with sides of length $a, b, c, d$ (in that order) and diagonals of length $p$ and $q$, we have $pq = ac + bd$.

**Summary** Ptolemy: In a convex quadrilateral with sides of length $a$, $b$, $c$, $d$ (in that order) and diagonals of length $p$ and $q$, we have $pq \le ac + bd$.

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