Proof Without Words: The Triple Angle Sine and Cosine Formulas

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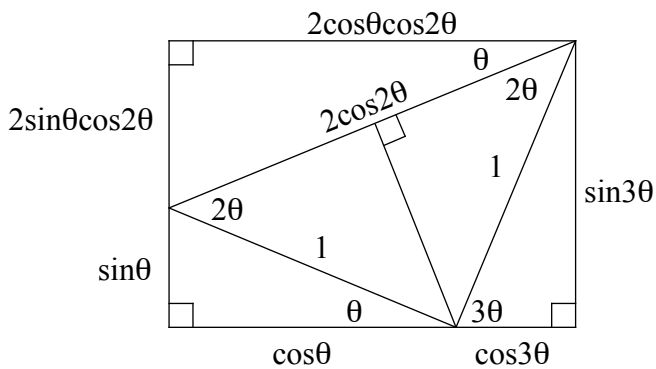

$$\begin{aligned}\sin 3\theta &= 2\sin\theta\cos 2\theta + \sin\theta \\ &= 2\sin\theta(1-2\sin^2\theta)+\sin\theta \\ &= 3\sin\theta - 4\sin^3\theta.\end{aligned}$$

$$\begin{aligned}\cos 3\theta &= 2\cos\theta\cos 2\theta - \cos\theta \\ &= 2\cos\theta(2\cos^2\theta - 1) - \cos\theta \\ &= 4\cos^3\theta - 3\cos\theta.\end{aligned}$$

REFERENCE

S. Okuda, Proof without words: The triple-angle formulas for sine and cosine. *Math. Mag.* **74** (2001), 135.

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